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*Published in:*  
Journal of Economics

*Publication date:*  
1993

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*  
Smulders, J. A., & Gradus, R. H. J. M. (1993). The trade-off between environmental care and long-term growth: Pollution in three proto-type growth models. *Journal of Economics*, 58(1), 25-51.

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## **The Trade-off Between Environmental Care and Long-term Growth — Pollution in Three Prototype Growth Models**

By

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(Received August 10, 1992; revised version received February 8, 1993)

The effects of increased environmental care on optimal technology choice and long-term growth are studied for an economy in which pollution is a side-product of physical capital used in production. First, it is shown that in case of a standard neoclassical production structure, the result is a less capital-intensive production process whereas the long-run growth rate is not affected. Next, we introduce assumptions of the endogenous growth literature. When there are constant returns to physical capital, an increase in abatement activities crowds out investment and lowers the endogenous growth rate. When human capital accumulation is the engine of growth, physical capital intensity declines and the endogenous optimal growth rate is unaffected by increased environmental care or is even higher, depending on whether or not pollution influences agents' ability to learn.

### **1. Introduction**

In recent years, during which an unpolluted environment has become more and more a scarce commodity, economists have shown an increasing interest in environmental issues. An important question in this

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\* The authors are grateful to an anonymous referee, to Lans Bovenberg, Noud Grujters, Theo van de Klundert, Lex Meijdam, and Sweder van Wijnbergen for helpful and stimulating comments. The views expressed in this paper are the authors' own and should not be attributed to the Ministry of Finance.



respect is whether the long-run growth rate of an economy is affected by environmental care. Some economists have argued that long-run growth rates depend on the efforts to clean up the environment, while others have argued that environmental efforts are necessary for the short run but will not influence long-term growth rates. In this paper we try to shed some light on the question by applying some of the insights of the new "endogenous growth" literature.

In the current literature on environmental issues in economics we can distinguish three main directions. The first analyzes how the intertemporal allocation of resources is affected by environmental issues. The classical Ramsey problem is applied to an economy where pollution is an inevitable side-effect of economic activity. Consumers maximize a utility function, which depends on both consumption and the quality of the environment, by choosing the level of investment, consumption, and abatement activities. Seminal work is carried out by Forster (1973) whose framework is extended by Gruver (1976), Luptacik and Schubert (1982), Siebert (1987), Van der Ploeg and Withagen (1991), and others. The main conclusion from this literature is that if one allows for pollution effects in the classical Ramsey problem, the optimal capital stock is less than under the golden rule.

A second direction in the literature focuses on the question, how a social optimum can be sustained in a market economy. Because pollution causes externalities, market outcomes are inefficient and there is a role for the government. Different instruments can be introduced like Pigouvian taxes, markets for pollution rights, binding quota restrictions, and property rights. The literature discusses the differences between these four instruments (e.g., Dasgupta, 1982; and Siebert, 1987). It is also possible to analyze this question within an optimal tax framework (e.g., Gradus and Kort, 1992), where the credibility of the government plays a role in the effectiveness of the different instruments.

In the third and more recent stream of literature the international aspects of pollution are analyzed. Hereby, different countries have different attitudes towards pollution, and cooperation may be important (e.g., Benhabib and Radner, 1989; Kaitala et al., 1991; and Van der Ploeg and De Zeeuw, 1992).

Surprisingly, long-term growth aspects of environmental economics are somewhat underexposed in the formal literature. In most models long-term growth is ignored or the growth rate is exogenously given (e.g., Mäler, 1975). In this paper we study the relations among pollution, technology choice, and optimal growth within three prototype growth models. We are especially interested in the question how these three entities are influenced if the society becomes more interested in environmental care.



In Sect. 2, we set up a basic structure for studying the problem. Pollution arises from the use of capital in production and enters the social welfare function as a disutility. Three model variants with different production structures are distinguished, corresponding to three representative growth models from the literature. First, we use the neo-classical production structure with exogenously given rate of technological progress. In Sect. 3, we turn to two main models from the endogenous growth literature. We consider the Romer (1986) production structure with constant returns to capital, which is popularized by Rebelo's (1991) "AK-model." It is shown that in this second model variant there is a negative relation between the optimal growth rate and the concern for the environment. As a third variant, we show that if the production structure of Lucas (1988) is applied, long-run growth is not influenced by environmental preferences. In Sect. 4, we extend the Lucas model (1988) further by assuming pollution effects on the production structure. Human capital accumulation is the engine of growth and the learning ability of people can be influenced by environmental issues. This model points out how an increased willingness to clean up pollution can stimulate growth. In Sect. 5, we reflect on the nature of environmental care by distinguishing between abatement of existing pollution and shifts towards less polluting production processes. Finally, Sect. 6 summarizes and gives some suggestions for future research. The appendix contains the full derivations of the model equations used in the text.

## 2. Optimal Growth Theory and Pollution

To study the effects of environmental care on optimal growth rates and technology choice, we consider an economy where pollution damages social welfare. Hereby, we build on the analysis originally invited by Forster (1973) and later on worked out in more details by Luptacik and Schubert (1982), and by Van der Ploeg and Withagen (1991). Pollution is an inevitable by-product of production, but can be diminished by devoting some part of output to abatement activities. As a result, the society faces a trade-off between consumption, growth, and abatement, all of which contribute to intertemporal welfare and the sum of which is constrained by the level of output.

In the literature there is some discussion about the source of pollution. Forster (1973), and Van der Ploeg and Withagen (1991) take pollution as a linear function of production. Luptacik and Schubert (1982) have three sources of pollution: consumption, production, and the capital stock. For the proposal of this paper we assume that the



amount of pollution is a function of the capital stock. Another important question in the literature is whether pollution should be modeled as a stock or a flow variable. In the earlier literature, e.g., Forster (1973) and Gruver (1976), the effects of the flow of pollution on welfare are considered. Luptacik and Schubert (1982) consider the effect of the stock on welfare, while Van der Ploeg and Withagen (1991) take both effects. Here we concentrate on pollution as a flow. However, it can be shown that production, rather than capital, as a source of pollution and/or pollution as a stock, rather than a flow, yield the same conclusions with respect to the relation between growth and environmental care.

Taking together our assumptions, the society's optimization problem can be written as

$$\max \int_0^{\infty} e^{-\vartheta t} U\left(\frac{C}{L}, P\right) dt, \quad \begin{aligned} &U_c > 0, \quad U_{cc} < 0, \quad U_P < 0, \\ &U_{PP} \leq 0, \quad U_{cP} \leq 0, \\ &U_{cc}U_{PP} - U_{cP}^2 \geq 0, \end{aligned} \quad (2.1)^1$$

$$\text{s. t. } P = P(K, A), \quad P_K > 0, \quad P_A < 0, \quad (2.2)$$

$$\dot{K} = Y(K, hL) - C - A, \quad K(0) = K_0, \quad (2.3)$$

where  $\vartheta$ ,  $c = C/L$ ,  $L$ ,  $P$ ,  $K$ , and  $A$  denote the discount rate, per capita consumption, population (work force), (net) pollution, physical capital, and abatement activities. A subscript denotes a partial derivative, a dot denotes a time derivative.  $Y$  represents output which is produced using physical capital ( $K$ ) and labor measured in efficiency units ( $hL$ ). For simplicity we ignore depreciation.

The social optimal plan implies two optimum conditions:

$$U_c = LU_P P_A, \quad (2.4)$$

$$\frac{\dot{c}}{c} = \left\{ \left( Y_K + \frac{U_{cP} \dot{P} + LU_P P_K}{U_c} \right) - (\vartheta + \lambda) \right\} \eta, \quad (2.5)$$

where  $\lambda \equiv \dot{L}/L$  and  $\eta \equiv -U_c/cU_{cc}$ , i.e., the elasticity of intertemporal substitution between current and future consumption. Equation (2.4) gives the optimum allocation between current consumption and current abatement activities. The marginal contributions to utility of both variables are equalized in the optimum. The marginal utility of abatement

<sup>1</sup> Note that social welfare is assumed to depend on the utility of a representative consumer over an infinite horizon. We do not discuss the effects of different weights on future generations or other intergenerational issues.



is multiplied by the size of the population ( $L$ ) because abatement affects pollution which has a public good character. Equation (2.5) gives the optimum allocation between current and future consumption. This allocation obviously depends on the marginal contribution to future utility of consumption foregone, which can be called the "social interest rate," denoted by  $r$ . Savings add to the current stock of capital and increase future output by  $Y_K$ . But future consumption is lower valued if pollution grows ( $U_{cP} \dot{P} \leq 0$ ). Moreover, a larger capital stock leads to increased pollution, which is a disutility ( $U_P P_K < 0$ ). Hence, the social interest rate  $r$  is represented by the first term in long brackets. It is optimal to postpone consumption ( $\dot{c}/c > 0$ ) when  $r$  exceeds the rate of time preference ( $\vartheta + \lambda$ ). Of course, Eq. (2.5) is a version of the well-known Keynes–Ramsey rule. It differs from the traditional version because of the wedge between the marginal product of capital ( $Y_K$ ) and the social marginal value of capital  $r$ .

The Ramsey rule for the decentralized economy is similar to Eq. (2.5), but with the term  $LU_P P_K / U_c$  left out. The market rate of interest is larger than the social rate of interest because the effects on pollution of an additional unit of capital are ignored by private agents. By introducing a tax on pollution, the social optimum can be attained.

Taking together both optimum conditions and defining  $r$  we can write

$$\frac{\dot{c}}{c} \left( = \frac{\dot{C}}{C} - \lambda \right) = \left\{ Y_K + \frac{P_K}{P_A} - \zeta \frac{\dot{P}}{P} - (\vartheta + \lambda) \right\} \eta, \quad (2.6)$$

$$r = Y_K + \frac{P_K}{P_A} - \zeta \frac{\dot{P}}{P}, \quad (2.7)$$

where  $\zeta \equiv -PU_{cP}/U_c$ . From these equations we see that the necessary condition for positive per capita growth is that the social rate of interest  $r$  may not fall below the rate of time preference  $\vartheta + \lambda$ . This imposes restrictions on the combination of the production function, the pollution function, and the utility function. With regard to the production function, the physical marginal product of capital,  $Y_K$ , must be sufficiently larger than the rate of time preference to compensate for the marginal abatement costs associated with capital ( $P_A/P_K < 0$ ) and the utility losses due to pollution ( $\zeta \dot{P}/P$ ). A falling marginal product of capital would decrease  $r$  below  $\vartheta + \lambda$  and growth would peter out. This condition on the production function can be met in several ways. In the sequel we consider three cases in all of which  $Y_K(K, hL)$  can be kept constant because  $hL$  grows, either exogenously or endogenously.



With regard to the pollution and abatement function, the net social cost of pollution  $|P_K/P_A|$  may not rise too fast relative to  $Y_K$ . If a larger and larger part of total resources  $Y$  has to be devoted to abatement when the capital stock rises, then at some moment in time it will become optimal to stop capital accumulation and growth. In other words, pollution may not technically be insurmountable. Therefore, in the subsequent growth analysis, we assume that pollution can remain constant when abatement activities are kept in pace with capital accumulation.

Finally  $\zeta \dot{P}/P$  may not rise too fast. If increasing pollution influences more and more the pleasure from consumption of physical goods (i.e., if  $\zeta$  increases in  $P$ ), the society will wish to stabilize or decrease pollution levels. Note that this tendency also results if one models absolute limits on the pollution level which can be borne by the environment.

We now turn to three specifications of the model being as close as possible to the main existing growth models. We choose specifications of the pollution and utility function that guarantee a balanced growth solution and we focus on long-term equilibria.

### 2.1 Neoclassical Model

In the traditional neoclassical (Cass–Koopmans) growth model, the production function  $Y(K, hL)$  is of the Solow type with constant returns to scale but diminishing returns to capital or labor separately. The supply of effective labor ( $hL$ ) is exogenous at any moment in time and grows at rate  $\nu + \lambda$ . In contrast with the interest rate, the long-run growth rate is solely determined by the technological opportunities and not by preferences or pollution. Changes in preferences affecting  $P_K/P_A$  and  $\zeta \dot{P}/P$  are offset by changes in  $Y_K$  such that  $C$  grows at an exogenous rate [see Eq. (2.6)]. As long as capital input is growing faster (slower) than effective labor input, the marginal product  $Y_K$  declines (rises) because of the diminishing returns to capital. Therefore, in a situation of balanced growth (with  $r$  constant), the rate of growth of capital, output, and (aggregate) consumption will equal the sum of the growth of the labor force,  $\lambda$ , and the rate of labor augmenting technological change,  $\nu$ , which are exogenously given. This growth rate is equal to the rate in the Cass–Koopmans economy without pollution.

To illustrate the described effects on growth ( $g$ ), marginal value of capital ( $r$ ), and physical marginal product ( $Y_K$ ) we take a Cobb–Douglas production function and choose the following, admittedly simple, specifications for utility and for pollution:



$$U(c, P) = \ln c - \frac{\phi}{1 + \psi} P^{1+\psi}, \quad (2.8)$$

$$P(K, A) = \left(\frac{K}{A}\right)^\gamma. \quad (2.9)$$

The social optimum is characterized by the following relations (see appendix for derivation):

$$g = \lambda + \nu, \quad (2.10)$$

$$g = r - \vartheta, \quad (2.11)$$

$$r = \beta \frac{Y}{K} - (\phi\gamma)^{1/\mu} \left[ \vartheta + (1 - \beta) \frac{Y}{K} \right]^{1/\mu}, \quad (2.12)$$

where  $\beta$  is the production elasticity of capital and  $\mu \equiv 1 + \gamma + \gamma\psi > 1$ . Equation (2.10) gives the growth rate that is attainable in the long run and is drawn in Fig. 1 by the line labeled TECH. Equation (2.11) is the Keynes–Ramsey rule for balanced growth [note that (2.8) implies  $\eta = 1$  and  $\zeta = 0$ ]. It is drawn as the PREF-line giving the growth rate associated to any social rate of return to saving and investment ( $r$ ) that is desired given intertemporal preferences. Finally, Eq. (2.12) defines the CAP-line giving for any  $r$  the output to capital ratio that is desired in the long run and that is consistent with optimal static allocation.

The solution for an economy where pollution effects of capital accumulation are not internalized (i.e., the Cass–Koopmans economy) is found by setting  $\phi = 0$ . Then, the marginal return to capital  $r$  is equal to the marginal product of capital  $\beta Y/K$ . Point M in Fig. 1 corresponds to this case. An interesting question is what will change, if the society's preferences are shifted towards more environmental care (i.e., a shift from  $\phi = 0$  to  $\phi_1$  or from  $\phi_1$  to  $\phi_2$ ). Long-run growth is not affected, because the growth rate in neoclassical growth theory is determined by technological parameters. What will change is, of course, the output to capital ratio. A larger disutility of pollution (larger  $\phi$ ) increases the wedge between marginal product and social marginal productivity of capital. To attain the same social rate of return on savings and investment, the marginal productivity of capital has to rise to offset the increased disutility of pollution and therefore the CAP-line shifts down. The economy will experience a transition period of lower growth during which the capital intensity declines. Due to the diminishing returns to capital the interest rate gradually recovers. In the new steady state the economy is transformed to a less capital intensive, less polluting production process and  $r$  and  $g$  are ultimately unchanged.



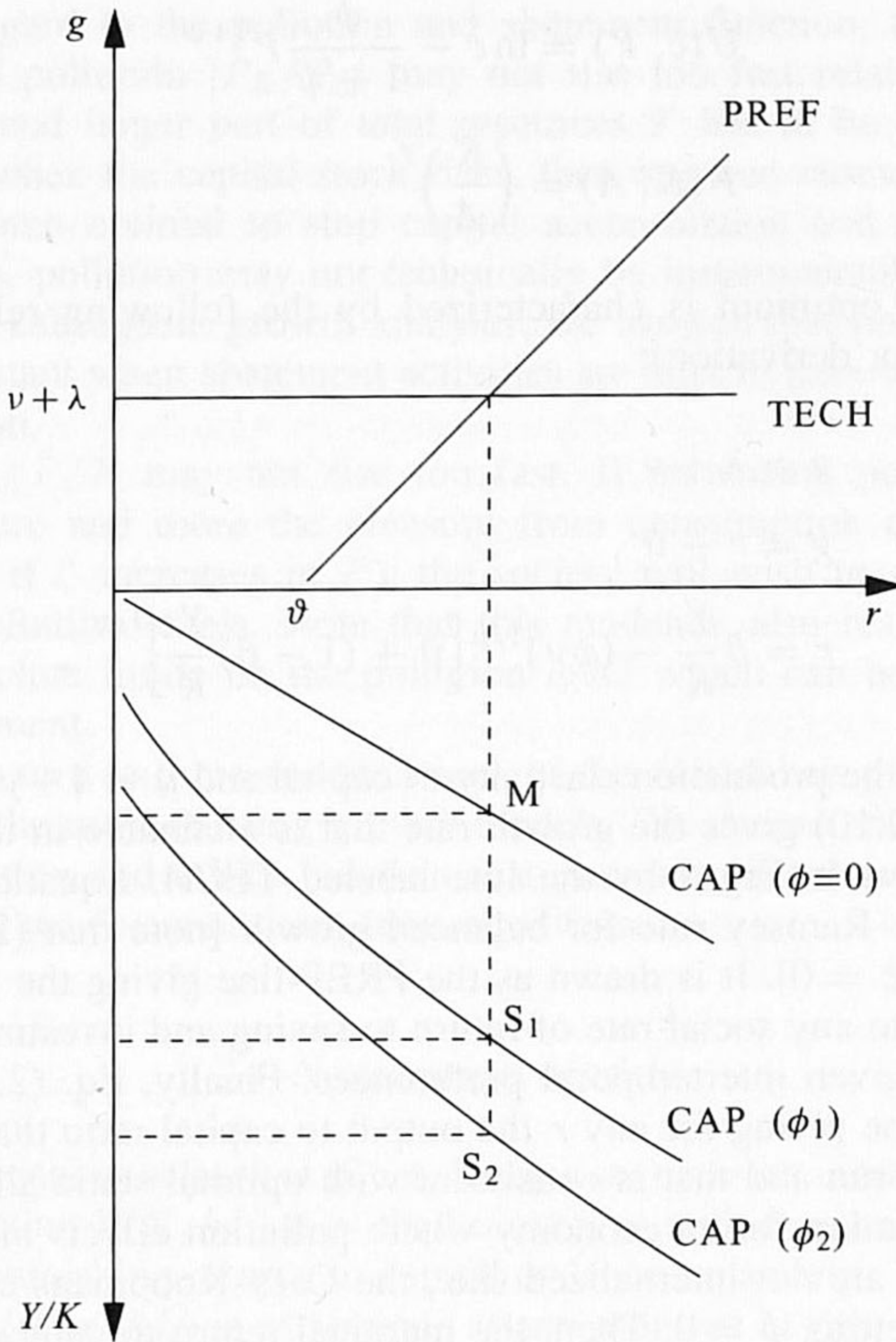


Fig. 1: Neoclassical case

### 3. Endogenous Growth

In the previous section it is described how environmental aspects can be incorporated in an optimizing framework. Using the neoclassical production structure, conclusions are drawn with respect to the optimal steady state *levels* (per effective labor unit) of economic variables, given the preference structure and the pollution process. To assess the effects of pollution and abatement on the *growth rate* of various variables, the neoclassical model is not suited, because it assumes rather than explains growth. In the steady state, growth is always at the exogenously given natural rate and changes in preferences only affect levels.



Recent models yield more flexible and in our opinion more satisfactory explanations for growth (e.g., Romer, 1986, 1990; Lucas, 1988; Grossman and Helpman, 1991; see Van de Klundert and Smulders, 1992, for a survey). In these “endogenous growth” models, technical change and accumulation of technical knowledge are the result of economic decisions regarding investment in physical or human capital and R&D activities. The production structure is therefore different from the production structure in the neoclassical growth model, where only physical capital can be accumulated subject to diminishing returns. In endogenous growth models each kind of capital can be accumulated. This gives rise to constant returns to a broad concept of capital including all reproducible factors of production. A faster rate of accumulation, due to for example a lower time preference, therefore does not imply falling marginal returns. This implies that a permanently higher rate of growth can be maintained at a higher but constant savings rate.

Within the “class” of endogenous growth models the distinction can be made between intentional and unintentional growth models. In the latter, the accumulation of growth-generating knowledge is an externality for economic agents. It arises rather mechanically as a side-product of investment (e.g., Romer, 1986). In the former, special efforts, resources, and activities have to be devoted to generate the knowledge needed for technical change and growth (e.g., Lucas, 1988; Grossman and Helpman, 1991). From both categories of models we will take one representative example and extend it to study some aspects of the relation between pollution and growth.

### 3.1 Rebelo Model

The simplest model to illustrate endogenous growth is Rebelo’s (1991) model where production takes place with capital  $K$  only according to:

$$Y = \alpha K . \quad (3.1)$$

Here  $\alpha$  is the marginal return to the stock of capital ( $K$ ) defined in a broad sense. This return  $\alpha$  is constant due to the fact that for example technical knowledge arises from investment and learning by doing, offsetting diminishing returns to capital in a narrow sense. The parameter  $\alpha$  can be dependent on the size of the economy measured by the working force  $L$  as is assumed in Romer (1986). In that specification  $L$  has to be constant to guarantee balanced growth, which we will assume in this section. The TECHNOLOGY line is no longer a flat line as in the previous section, but a vertical line at  $\alpha$ . The equilibrium rate of growth



is found at the intersection of the technology line and the PReFerences line which is again the Ramsey formula. A decline in time preference shifts the preference line to the left and the growth rate is permanently higher.

Environmental issues can be incorporated in the same way as in the preceding section by assuming a disutility of pollution which is a by-product of the use of capital in production. The social optimum is found by maximizing social welfare function (2.1) subject to pollution function (2.2) and goods market equilibrium (2.3) with  $Y(K, hL)$  replaced by  $Y = \alpha K$  as in (3.1). This yields

$$\frac{\dot{c}}{c} \left( = \frac{\dot{C}}{C} - \lambda \right) = \left\{ \alpha + \frac{P_K}{P_A} - \zeta \frac{\dot{P}}{P} - (\vartheta + \lambda) \right\} \eta. \quad (3.2)$$

Comparing this result with the neoclassical case, the endogenous marginal product of capital  $Y_K$  is replaced by the exogenous  $\alpha$ , and the growth rate is endogenous. Hence the wedge  $(P_K/P_A - \zeta \dot{P}/P)$  and the growth rate  $g$  adjust to shifts in parameters. A rise in the preference for a clean environment will widen the wedge between the marginal product of capital and the social value of capital  $|P_K/P_A| + \zeta \dot{P}/P$ , and growth will be lower.

To set up an illustrative figure like in the previous section we choose again specifications (2.8) and (2.9) for utility and for pollution. In the appendix we show that then the social optimum is characterized by the following relations:

$$g = r - \frac{1}{\phi\gamma}(\alpha - r)^\mu, \quad \text{TECH-line,} \quad (3.3)$$

$$g = r - \vartheta, \quad \text{PREF-line,} \quad (3.4)$$

$$\frac{Y}{K} = \alpha, \quad \text{CAP-line,} \quad (3.5)$$

where  $r$  is, as before, the marginal social value of capital. Relation (3.4) is again the Ramsey formula or PREF-line. Relation (3.3) defines the TECH-line which gives the growth rate that is sustainable for any  $r$  in the long run and consistent with optimal static allocation (of production over consumption, abatement, and investment). To understand why this line is upward sloping, notice that a higher rate of investment ( $g$ ) crowds out consumption and abatement activities (per unit of capital). When as a consequence a smaller part of the physical returns to capital has to be spent on abatement, the return to capital  $r$  is larger.<sup>2</sup> Figure 2 depicts the three relations.



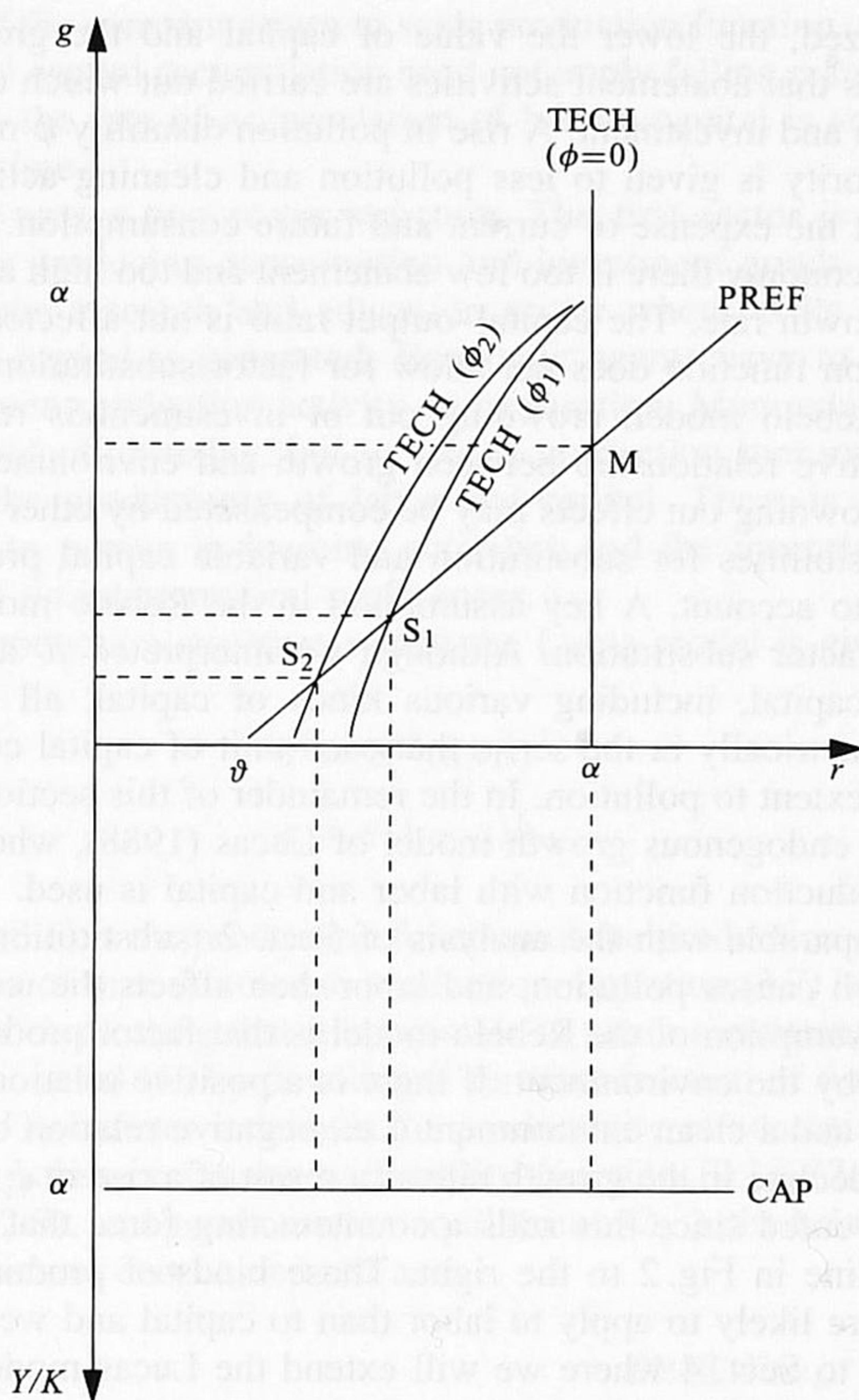


Fig. 2: "Rebelo case"

Point M corresponds to the case in which pollution effects are not internalized (or  $\phi = 0$ ), point S<sub>1</sub> (S<sub>2</sub>) to the optimum for  $\phi = \phi_1$  ( $\phi = \phi_2$ ). The solution for  $\phi = 0$  can be interpreted as a market solution where no pollution effects are internalized. The more pollution effects

<sup>2</sup> From goods market equilibrium (2.3) follows:  $(C + A)/K = (Y - \dot{K})/K = \alpha - g$ : increased investment crowds out  $(C + A)/K$  and it is optimal to spread this burden by reducing both  $C/K$  and  $A/K$ . Since the return to capital is  $r = Y_K + P_K/P_A = \alpha - A/K$  [cf. (2.7) and (2.9)] reducing  $A/K$  means increasing  $r$ .



are internalized, the lower the value of capital and the growth rate. The reason is that abatement activities are carried out which crowd out consumption and investment. A rise in pollution disutility  $\phi$  means that a higher priority is given to less pollution and cleaning activities are intensified at the expense of current and future consumption.<sup>3</sup> Thus, in the market economy there is too few abatement and too high a pollution level and growth rate. The capital–output ratio is not affected because the production function does not allow for factor substitution.

In the Rebelo model, crowding out of investment is responsible for the negative relationship between growth and environmental care. However, crowding out effects may be compensated by other effects as soon as possibilities for substitution and variable capital productivity are taken into account. A key assumption in the Rebelo model is the absence of factor substitution. Although we interpreted  $K$  as a broad concept of capital, including various kinds of capital, all kinds are treated symmetrically in the sense that each unit of capital contributes to the same extent to pollution. In the remainder of this section we will consider the endogenous growth model of Lucas (1988), where a neo-classical production function with labor and capital is used. That case is more comparable with the analysis of Sect. 2: substitution between capital, which causes pollution, and labor then affects the results. Another key assumption of the Rebelo model is that factor productivity is not affected by the environment. If there is a positive relation between productivity and a clean environment (i.e., negative relation between  $\alpha$  and  $P$ ), the decline in the growth rate as a result of a rise in  $\phi$  is smaller or even prevented since this adds a counteracting force that shifts the technology line in Fig. 2 to the right. These kinds of productivity effects are more likely to apply to labor than to capital and we postpone a discussion to Sect. 4 where we will extend the Lucas model.

### 3.2 Lucas Model

In Lucas (1988), production takes place according to a neoclassical production function (which allows us to reintroduce population growth). However, not only physical capital can be accumulated but also the skills of labor or human capital. In the economy as a whole there are constant returns with respect to all factors (physical and human capital) taken together (i.e., with respect to the broad concept of capital),

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<sup>3</sup> The new equilibrium is reached without transitional dynamics since there is only one predetermined state variable ( $K$ ).



because of the constant return to scale production function. A faster rate of physical capital accumulation need not imply falling marginal returns as long as the rate of accumulation of human capital is accelerated in the same time.

Lucas uses a two sector structure. The first sector is the production sector producing consumption and investment goods. The second sector is the research and education sector where skills, knowledge, or human capital is generated. Economic agents have to divide their time between production activity and education. More education today lowers production today, but increases production tomorrow, because it raises the productivity of labor and capital. There is therefore an incentive to pursue in learning activities and the incentive is clearly dependent on intertemporal preferences.

The production structure within the Lucas model is given by:

$$Y = K^\beta (uhL)^{1-\beta} , \quad (3.6)$$

$$\dot{h} = \epsilon(1 - u)h . \quad (3.7)$$

Equation (3.6) is the production function of the production sector with  $u$  the fraction of time devoted to production. Equation (3.7) is the "Engine of Growth" indicating that it is possible to attain a constant growth rate of human capital ( $h$ ) by devoting a constant fraction of time ( $1 - u$ ) to education. Parameter  $\epsilon$  denotes the productivity of education activities. Combining these relations with utility function (2.1), (2.8), pollution function (2.9), and goods market equilibrium (2.3), the balanced growth social optimum can be derived as

$$g = r - \vartheta , \quad \text{PREF-line,} \quad (3.8)$$

$$r = \epsilon + \lambda , \quad \text{TECH-line,} \quad (3.9)$$

$$r = \beta \frac{Y}{K} - (\phi\gamma)^{1/\mu} \left[ \vartheta + (1 - \beta) \frac{Y}{K} \right]^{1/\mu} , \quad \text{CAP-line,} \quad (3.10)$$

where  $r$  is again the marginal value of capital. The intertemporal preference relation between  $r$  and  $g$  (3.8) and the relation between the desired capital intensity and  $r$  (3.10) are the same as in the neoclassical case because they are derived independently of the assumptions regarding the growth of human capital. Equation (3.9) is the technology line which is a vertical line. It can be interpreted as an arbitrage condition stating that the marginal returns to investment in physical capital  $r$  equal the marginal returns to investment in human capital. The latter are the sum of the exogenous marginal productivity in the engine of



growth sector ( $\epsilon$ ) and the growth rate of the labor force which benefits from the knowledge generated in the education sector ( $\lambda$ ).

The three lines are depicted in Fig. 3. The upper part of the figure is exactly the same as in Lucas' original formulation without pollution. Per capita growth is stimulated by a decline in time preference (preference line shifts to the left) and by a rise in the productivity in the education sector (it becomes attractive to spend more time on education, human capital grows faster raising the marginal productivity of physical capital and invoking a higher rate of accumulation of physical capital, too). Environmental preferences do not matter for the growth rate or "interest rate," but only affect the capital intensity. The reason is that internalizing pollution influences only the marginal value of *physical* capital and not of human capital. Arbitrage between human capital accumulation and physical capital accumulation ensures that the rate of return to physical capital equals the exogenous rate of return to human capital. A rise in  $\phi$  lowers initially the social value of physical capital, physical investment is slowed down and the capital intensity in the production sector declines in such a way that the increased marginal product of physical capital offsets the increased disutility of pollution through capital. In the lower part of Fig. 3 this is illustrated by drawing Eq. (3.10).

The market solution can be mimicked by setting  $\phi = 0$ . Growth will be at the same rate as in the social optimum but the capital intensity will be too high. The reason is of course that the pollution effect of capital accumulation is not internalized.

#### 4. Health, Pollution, and Growth

So far, we have studied only the implications of internalizing the direct negative aspects of pollution on social welfare. In fact, pollution can also change production opportunities, e.g., by reducing the quality of natural inputs or by increasing the deterioration of physical equipment. In this section we extend the Lucas (1988) model in the sense that we allow for effects of pollution on the marginal returns to education. The idea is that pollution affects health of workers, which lowers their ability to learn. Empirical support for an example of such a relation between pollution and human capital formation is found in Margulis (1991). He first reports the empirically significant correlation between lead in air and blood lead levels. Next, he shows that children with higher blood lead levels have a lower cognitive development and require supplemental education.

We assume that more pollution, e.g., in the form of smog, air pol-



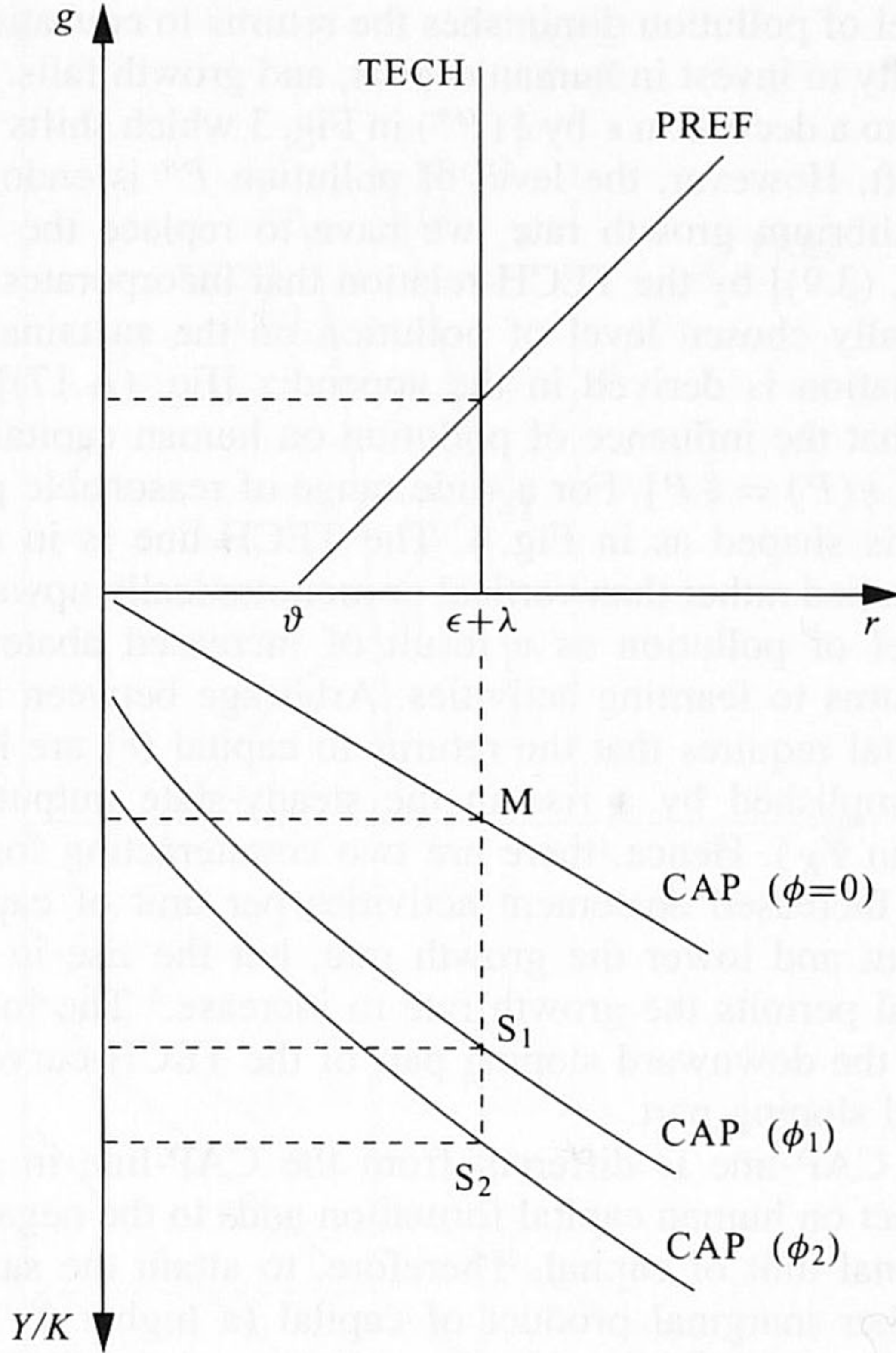


Fig. 3: "Lucas case"

lution, ground pollution, and nuclear pollution, causes human capital to depreciate at a faster rate such that we can replace (3.7) in the extended Lucas model of the previous section by

$$\dot{h} = \{\epsilon(1 - u) - \xi(P)\}h, \quad \xi'(P) > 0, \quad (4.1)$$

where  $\xi(P)$  represents the influence of pollution on the learning process. The Keynes–Ramsey rule in the situation of balanced growth now reads

$$g - \lambda = \{(\epsilon + \lambda - \xi(P^*)) - (\vartheta + \lambda)\}, \quad (4.2)$$

where  $P^*$  is the steady state level of pollution. Compared with the previous case, the social marginal value of capital is reduced by  $\xi(P^*)$ .



A higher level of pollution diminishes the returns to education, reduces the profitability to invest in human capital, and growth falls. This effect is equivalent to a decline in  $\epsilon$  by  $\xi(P^*)$  in Fig. 3 which shifts the TECH-line to the left. However, the level of pollution  $P^*$  is endogenous. To find the equilibrium growth rate, we have to replace the TECH-line of Fig. 3 [Eq. (3.9)] by the TECH-relation that incorporates the effects of the optimally chosen level of pollution on the sustainable growth rate. This relation is derived in the appendix [Eq. (A.17)] under the assumption that the influence of pollution on human capital formation is linear [i.e.,  $\xi(P) = \bar{\xi}P$ ]. For a wide range of reasonable parameters, this relation is shaped as in Fig. 4. The TECH-line is in the present case hump-shaped rather than vertical or monotonically upward sloping. A lower level of pollution as a result of increased abatement leads to higher returns to learning activities. Arbitrage between human and physical capital requires that the returns to capital ( $r$ ) are higher, too. This is accomplished by a rise in the steady-state output to capital ratio (a rise in  $Y_K$ ). Hence, there are two counteracting forces on the growth rate. Increased abatement activities per unit of capital crowd out investment and lower the growth rate, but the rise in output per unit of capital permits the growth rate to increase.<sup>4</sup> The former effect dominates at the downward sloping part of the TECH-curve, the latter at the upward sloping part.

Also the CAP-line is different from the CAP-line in Fig. 3. The pollution effect on human capital formation adds to the negative effects of an additional unit of capital. Therefore, to attain the same interest rate  $r$ , a higher marginal product of capital (a higher  $Y/K$  ratio) is desired to offset the stronger negative effects and the CAP-line shifts down compared with the situation in Fig. 3.

Compared with the "Rebelo case," the effect of a change in environmental preferences on the growth rate is reversed. A rise in  $\phi$  shifts the TECH-line upward and the CAP-line downward. The equilibrium growth rate is higher: more cleaning activities are preferred and investment becomes more attractive. Investment can be higher because a rise in the output to capital ratio prevents crowding-out effects. In a market economy (with a solution mimicked by a very low value of  $\phi$ <sup>5</sup>) the low

<sup>4</sup> Cf. footnote 2. Here, however,  $Y_K$  is given by  $\beta Y/K$  instead of  $\alpha$ .

<sup>5</sup> In contrast with the previous sections, the choice of the pollution function does not allow to interpret the case where  $\phi = 0$  as the market solution because this would imply an infinite level of pollution and negative growth. Of course, this is not a realistic case. Especially when the level of pollution is very high, firms will internalize some of the pollution effects. The outcome will be equivalent to the model solution for a small value of  $\phi$ .



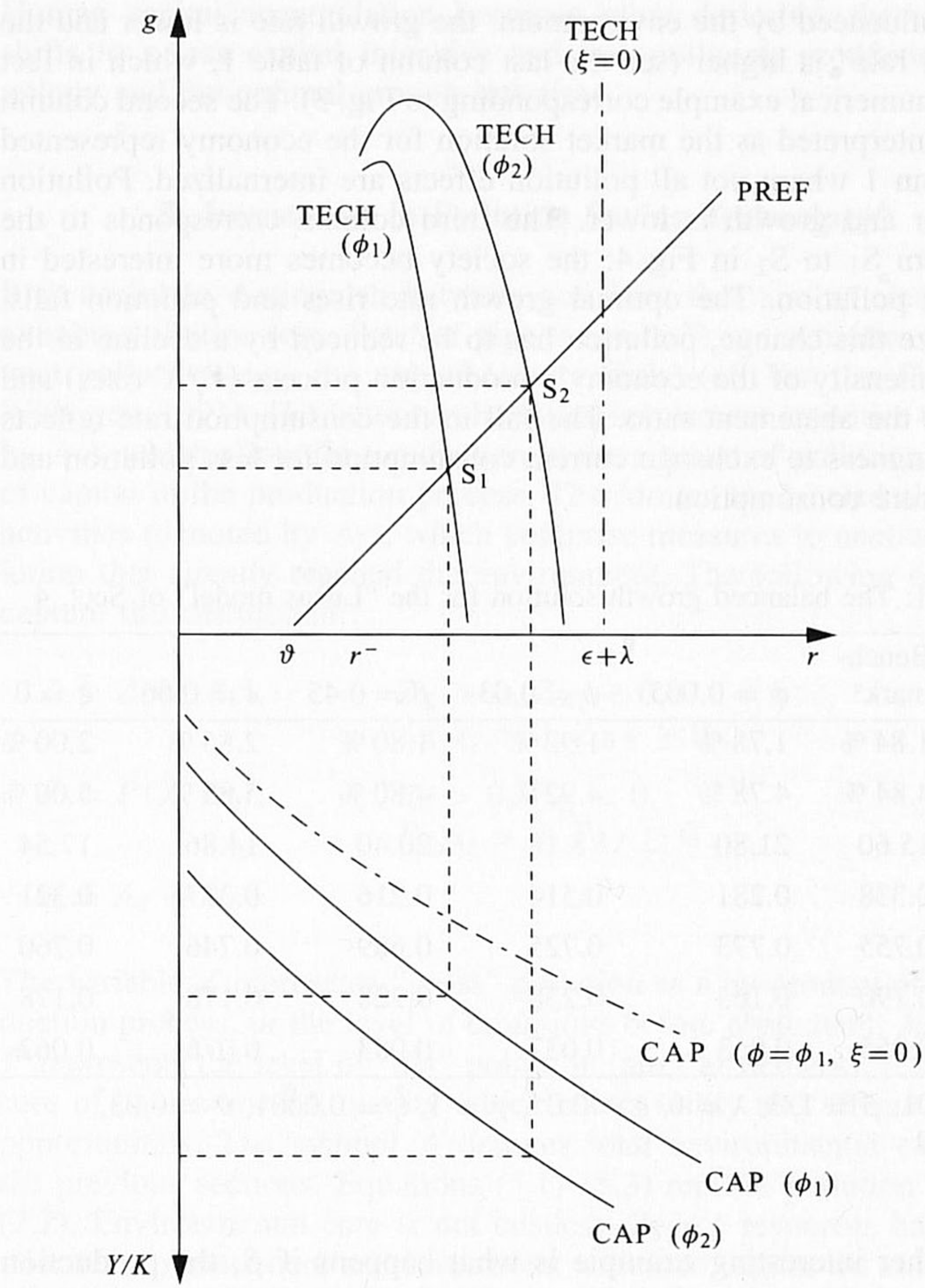


Fig. 4: Lucas case with health/education effects

private incentives to relieve the society's pollution problem result in a growth rate that is less than optimal.

The nature of the solution examined may be further clarified by a numerical example. In the first column of table 1 the social optimum for reasonable benchmark parameter values is given. If we compare these results with the model of Sect. 3 where human capital formation



is not influenced by the environment, the growth rate is lower and the “interest rate” is higher (see the last column of table 1, which in fact gives a numerical example corresponding to Fig. 3). The second column can be interpreted as the market solution for the economy represented in column 1 where not all pollution effects are internalized. Pollution is higher and growth is lower. The third column corresponds to the shift from  $S_1$  to  $S_2$  in Fig. 4: the society becomes more interested in cleaning pollution. The optimal growth rate rises and pollution falls. To realize this change, pollution has to be reduced by a decline in the capital intensity of the economy’s production process ( $Y/K$  rises) and a rise in the abatement ratio. The fall in the consumption rate reflects the willingness to exchange current consumption for less pollution and more future consumption.

Table 1: The balanced growth solution for the “Lucas model” of Sect. 4

	Bench- mark <sup>a</sup>	$\phi = 0.005$	$\phi = 0.03$	$\beta = 0.45$	$\epsilon = 0.06$	$\bar{\xi} = 0$
$g$	1.84 %	1.78 %	1.92 %	1.80 %	2.85 %	2.00 %
$r$	4.84 %	4.78 %	4.92 %	4.80 %	5.85 %	5.00 %
$P$	15.60	21.80	8.18	20.40	14.86	17.54
$Y/K$	0.338	0.281	0.514	0.216	0.377	0.321
$C/Y$	0.755	0.773	0.725	0.689	0.746	0.760
$A/Y$	0.190	0.163	0.238	0.228	0.178	0.178
$\dot{K}/Y$	0.055	0.063	0.037	0.083	0.076	0.062

<sup>a</sup>  $\phi = 0.01, \beta = 1/3, \lambda = 0, \epsilon = 0.05, \gamma = 1, \bar{\xi} = 0.0001, \vartheta = 0.03, \psi = 0.1$

Another interesting example is what happens if  $\beta$ , the production elasticity of physical capital, increases (column 4). As the productivity of capital rises relative to the productivity of labor input, it is optimal to shift to a more capital intensive production process. This requires larger spending on abatement to mitigate the pollution effects ( $A/Y$  rises). This, however, does not fully prevent pollution to rise and growth to slow down. Within this setting, an economy that is heavily dependent on relatively capital intensive sectors of industry has to finance large cleaning activities and to accept a lower growth rate.

If  $\epsilon$  increases we have the reverse: the economy described in column 5 has a comparative advantage in activities with high learning potential relative to the economies described in the preceding columns.



Human capital accumulation becomes more desirable, the economy shifts to a less capital intensive and less polluting production technology and the optimal growth rate rises.

### 5. Investment in Pollution Saving Technology<sup>6</sup>

It is useful to distinguish between activities that aim at cleaning up existing pollution (e.g., "end-of-pipe measures") and activities that prevent pollution (e.g., the use of cleaner fuels) (cf. Van der Ploeg and Withagen, 1991). The latter can be called investment activities (denoted by  $A_1$ ), which allow for a reduction in the amount of pollution per unit of capital in the production process. The former are labeled abatement activities (denoted by  $A_2$ ), which comprise measures to neutralize pollution that already reached the environment. The following equations capture this distinction<sup>7</sup>:

$$Z = Z(K, A_1), \quad \begin{aligned} Z_K &> 0, \quad Z_{A_1} < 0, \\ Z_{A_1 A_1} &> 0, \quad Z_{KK} \geq 0, \end{aligned} \quad (5.1)$$

$$P = P(Z, A_2), \quad \begin{aligned} P_Z &> 0, \quad P_{A_2} < 0, \\ P_{A_2 A_2} &> 0, \quad P_{ZZ} \geq 0, \end{aligned} \quad (5.2)$$

$$A = A_1 + A_2. \quad (5.3)$$

The variable  $Z$  represents "gross" pollution as a by-product of the production process, or the level of emissions before abatement. As before,  $P$  represents the level of "net" pollution (after abatement). It is a measure of environmental quality which enters utility and affects learning opportunities. The symbol  $A$  denotes total environmental care as in the previous sections. Equations (5.1)–(5.3) replace pollution function (2.2). Environmental care is not costless. Scarce resources have to be allocated to abatement and investment activities. Again we assume that the resources needed for environmental care ( $A$ ) can be produced in the

<sup>6</sup> Our discussion of pollution-saving investment is motivated by the suggestion of one of the referees.

<sup>7</sup> Alternatively, emissions ( $Z$ ) may depend on accumulated knowledge about cleaner technology rather than on current investment spending (cf. Van der Ploeg and Withagen, 1991, p. 230). If knowledge is produced in the  $Y$ -sector, this complicates the model without affecting the basic conclusions for the steady state. A growing economy requires a growing stock of pollution-saving knowledge to keep pollution constant and this asks for investment spending each moment in time.



goods sector of the economy [see goods market equilibrium, Eq. (2.3)]. The optimal allocation of scarce resources over abatement and investment requires that the marginal revenues of both activities are equal:

$$P_Z Z_{A_1} = P_{A_2} . \quad (5.4)$$

Since investment and abatement goods are produced with the same technology and since both affect the economy through net pollution, the optimal abatement–investment ratio depends only on characteristics of the pollution functions (5.1) and (5.2). The decision with regard to *total* spending on environmental care depends on preferences for environmental quality and on the impact of environmental quality on production and learning opportunities as in the model variants above. This decision can be analyzed separately from the decision regarding the abatement investment ratio, which is in fact already done in the previous part of this paper. According to (5.4), investment will be large, relative to abatement, if the pollution intensity of the production process can be easily reduced by investment ( $|Z_{A_1}|$  large for small  $A_1$ ), if the impact of pollution emissions on environmental quality is large ( $P_Z$  large for large  $Z$ ) and if abatement is difficult ( $|P_{A_2}|$  small for large  $A_2$ ).

As a simple illustration, the pollution functions are specified as:

$$Z = K^\gamma A_1^{-\gamma_1} , \quad (5.5)$$

$$P = Z A_2^{-\gamma_2} . \quad (5.6)$$

To focus on a steady state where growth is balanced and sustainable (i.e., pollution is constant in the steady state), it is assumed that  $\gamma_1 + \gamma_2 = \gamma$ . In a situation where polluting capital ( $K$ ) and abatement ( $A_2$ ) grow at rate  $g$ , pollution increases at rate  $(\gamma - \gamma_2)g$ . Emission reducing investment activities, that grow also at rate  $g$ , are needed to keep pollution constant. With the constant elasticity specifications (5.5) and (5.6), the optimal investment–abatement ratio reads:

$$\frac{A_1}{A_2} = \frac{\gamma_1}{\gamma_2} . \quad (5.7)$$

Substituting (5.3) and (5.5)–(5.7) yields the following semi-reduced form for net pollution:

$$P = [\gamma^\gamma \gamma_1^{-\gamma_1} \gamma_2^{-\gamma_2}] \left( \frac{K}{A} \right)^\gamma . \quad (5.8)$$



Note that (5.8) is equal to the specification of the pollution function in the previous sections, apart from the constants in square brackets. Hence, all conclusions with respect to the relationship between environmental care and growth derived before carry over to the model where environmental care takes the form of both abatement and investment.

## 6. Conclusions

In this paper we investigate whether the optimal long-run rate of growth is affected if society's preferences shift towards a larger concern for a clean environment. The answer to this question depends critically on the assumptions regarding production technology and the relation between pollution, production, and abatement. We focus on different assumptions concerning production technology, while we assume that preferences and technology are such that sustainable development with positive growth is possible.

One case we study is that of a standard neoclassical production function with substitution possibilities between the factors of production that cause pollution (capital) and the factors that cause no pollution (skilled labor). Then, the preference shift results in a production process that uses less intensively the polluting factor. We show that the economy can sustain the pre-shift growth rate because of (at least) two reasons. First, in the exogenous growth model, one factor of production (skilled labor) grows at an exogenous rate and only this rate determines the natural growth rate of the economy. Second, and more interestingly, in an endogenous growth model, each factor of production can be accumulated and long-run growth remains unchanged if the productivity of the growth generating activity is not affected by any change in pollution. We showed this in Sect. 3 for the "Lucas case" where human capital accumulation is the "engine of growth."

Other assumptions on production structure can give rise to a relation between optimal growth and environmental care. In Sect. 3, we considered an endogenous growth model along the lines of Romer (1986) and Rebelo (1991). No transformation to a less polluting production process is possible and all factors of production contribute equally to pollution. The optimal rate of growth will be lower if pollution is more disliked because increased abatement activities crowd out investment. This supports the rather pessimistic view that there may be a negative relation between growth and environmental care. However, as soon as there are possibilities of substitution towards less polluting production processes, the crowding out effect may be dominated by substitution effects.



In the different and perhaps more realistic endogenous growth model presented in Sect. 4, the productivity of the engine of growth is stimulated by a cleaner environment. In this case, the optimal long-run growth rate is higher the more society is ready to devote resources to clean up pollution. Less resources are available for capital accumulation (crowding out effect) but this is offset by the fact that both physical and human capital formation are more productive. Increased environmental care reduces pollution which is an argument in the "engine of growth" function and hence growth is affected. This result is similar to Rebelo (1991) where it is shown that taxes affect growth when they fall on the sectors of the economy that act as the engine of growth.

The assumptions on the pollution process are important for reaching these conclusions. In Sect. 2, we discussed necessary conditions for long-term growth with pollution. Of course, a lot of other specifications of production structure and pollution processes, which may depart more heavily from standard growth models than the specification put forward here, can be elaborated. Taking production, rather than physical capital, as the source of pollution does not change the main conclusions. For the Rebelo model this is trivial, since production depends only on physical capital. For the neoclassical and Lucas variants, the intuition is that the basic mechanisms, viz., crowding out of investment, productivity gains from a cleaner environment, and factor substitution, all remain present. Also taking pollution as a stock, rather than a flow, will not change the model dramatically. In a steady state with a constant stock of pollution, the flow of pollution will equal the absorption capacity of the environment which is (negatively) dependent on the existing stock of pollution. Hence, the flow of pollution and the stock of pollution are directly related.

Other relations between growth and environmental issues are still to be worked out. The most obvious from the point of view of the endogenous growth literature is to extend the Romer (1990) or Grossman and Helpman (1991) model. They argue that it is necessary to devote activities and resources to a research sector to generate the knowledge that is necessary for growth. The model resembles very much the Lucas (1988) model (which is the reason why we did not explore it here), but now research and development is the engine of growth rather than human capital formation. Analogous to this model one could argue that there is a need for special R&D activities aimed at the development of cleaner production methods, dissolvable plastics, efficient abatement technology, etc. A shift in preferences for the environment will cause a reallocation between the production sector and the R&D sectors. Our guess is that if the increase in the size of the environmental R&D sector crowds out other R&D activities, growth can be hurt.



Growth issues are interesting in the two other directions within the current environmental economics literature mentioned in the introduction. This regards first the choice of instruments to attain optimum growth in a decentralized economy, which is especially interesting in case pollution itself cannot be measured or taxed. Secondly, it is worthwhile to consider international interdependence, environmental spillovers, and growth.

### Appendix

The symbols are defined as in the text, unless otherwise indicated. To derive the TECH, PREF, and CAP relations, social welfare [Eq. (2.1)] is maximized with respect to  $C$ ,  $A$ , and  $u$ , subject to (2.3) (goods market equilibrium), (2.8) (instantaneous utility), (2.9) (pollution function), and

$$Y = \alpha K^\beta (uhL)^{1-\beta}, \quad \text{production function,} \quad (\text{A.1})$$

$$\hat{h} \equiv \frac{\dot{h}}{h} = \epsilon(1-u) - \xi P + v, \quad \text{engine of growth,} \quad (\text{A.2})$$

$$0 \leq u \leq 1, \quad \text{time constraint.} \quad (\text{A.3})$$

The essential parametrical restrictions for the neoclassical model are  $0 < \beta < 1$ ,  $\epsilon = 0$ ,  $\xi = 0$ , for the “Rebelo model”  $\beta = 1$ , and for the “Lucas model”  $0 < \beta < 1$ . For convenience we normalize  $\alpha = 1$  in the neoclassical and Lucas model and we set  $v = 0$  in the “Lucas model.” Moreover, with  $\epsilon = 0$ ,  $u$  will be chosen at its maximum value so that  $u$  can be treated as a parameter in the neoclassical case with  $u = 1$ . For the “Lucas case” we restrict the analysis to combinations of parameter values for which the restrictions in (A.3) are never binding. To abbreviate notation we define  $\mu \equiv 1 + \gamma + \gamma\psi$  and write  $\hat{x}$  for the growth rate ( $\dot{x}/x$ ) of any variable  $x$  (so  $\lambda = \hat{L}$ ). The present value Hamiltonian is

$$H = e^{-\vartheta t} \left\{ \ln \frac{C}{L} - \frac{\phi}{1+\psi} \left( \frac{A}{K} \right)^{-\gamma(1+\psi)} + \right. \\ \left. + \Theta_1 (\alpha K^\beta (uhL)^{1-\beta} - C - A) + \right. \\ \left. + \Theta_2 \left( \epsilon(1-u) - \xi \left( \frac{A}{K} \right)^{-\gamma} + v \right) h \right\}.$$



The *optimum conditions* read (after some slight rewriting to facilitate later substitutions):

$$\frac{\partial H}{\partial C} = \frac{1}{C} - \Theta_1 = 0, \quad (\text{A.4})$$

$$\frac{\partial H}{\partial A} = \left( \phi \gamma \left( \frac{A}{K} \right)^{-\mu} + (\Theta_2 h) \gamma \xi \left( \frac{A}{K} \right)^{\psi \gamma - \mu} \right) K^{-1} - \Theta_1 = 0, \quad (\text{A.5})$$

$$\frac{\partial H}{\partial u} = (1 - \beta)(\Theta_1 K) \frac{Y/K}{u} - \epsilon(\Theta_2 h) = 0, \quad (\text{A.6})$$

$$\begin{aligned} \frac{\partial H}{\partial K} &= \Theta_1 \beta \frac{Y}{K} - \left( \phi \gamma \left( \frac{A}{K} \right)^{-\mu} + (\Theta_2 h) \gamma \xi \left( \frac{A}{K} \right)^{\psi \gamma - \mu} \right) \frac{A/K}{K} \\ &= \vartheta \Theta_1 - \dot{\Theta}_1, \end{aligned} \quad (\text{A.7})$$

$$\frac{\partial H}{\partial h} = (1 - \beta)(\Theta_1 K) \frac{Y/K}{h} + \Theta_2 \hat{h} = \vartheta \Theta_2 - \dot{\Theta}_2, \quad (\text{A.8})$$

where  $\Theta_1$  and  $\Theta_2$  are the shadow prices associated to physical and human capital, respectively. Variable  $r$  is defined as the marginal social value of capital in terms of utility:

$$r = \frac{1}{\Theta_1} \frac{\partial H}{\partial K}.$$

Using (A.7), (A.5), and (A.4) this can be written as

$$r = \beta \frac{Y}{K} - \frac{A}{K} (= \vartheta - \hat{\Theta}_1 = \vartheta + \hat{C}). \quad (\text{A.9})$$

The *conditions for balanced growth*, i.e., constant growth rates and constant allocation, are

$$\hat{Y} = \hat{C} = \hat{A} = \hat{K} \equiv g, \quad \text{allocation in goods market}, \quad (\text{A.10})$$

$$\hat{h} + \hat{L} = \hat{K}, \quad Y/K \text{ constant}, \quad (\text{A.11})$$

$$-\hat{\Theta}_1 = \hat{K}, \quad -\hat{\Theta}_2 = \hat{h}, \quad \Theta_1 K, \Theta_2 h \text{ constant}. \quad (\text{A.12})$$

Condition (A.10) follows from (A.9), then (A.11) follows from (A.1) and (A.12) follows from (A.5), (A.8), and (A.10). To eliminate  $(\Theta_2 h)$  use (A.8) and (A.12) to get<sup>8</sup>  $\Theta_2 h = (1 - \beta)(\Theta_1 K)(Y/K)/\vartheta$ , which



yields, together with (A.5) and (A.4),

$$\frac{1}{\Theta_1 K} = \frac{1}{\phi\gamma} \left( \left( \frac{A}{K} \right)^\mu - \xi(1-\beta) \frac{\gamma}{\vartheta} \left( \frac{Y}{K} \right) \left( \frac{A}{K} \right)^{\psi\gamma} \right) = \frac{C}{K} . \quad (\text{A.13})$$

Now use (2.3), (A.13), and (A.10) to derive (A.14); and use (A.7), (A.5), and (A.12) to derive (A.15):

$$\frac{Y}{K} = \frac{1}{\phi\gamma} \left( \left( \frac{A}{K} \right)^\mu - \xi(1-\beta) \frac{\gamma}{\vartheta} \left( \frac{Y}{K} \right) \left( \frac{A}{K} \right)^{\psi\gamma} \right) + \frac{A}{K} + g , \quad (\text{A.14})$$

$$r = \vartheta + g . \quad (\text{A.15})$$

Equation (A.15) is the PREF-relation. Equations (A.9), (A.14), and (A.15) can be reduced to a relation between the variables  $r$  and  $Y/K$  which is the CAP-relation. Note that if  $\xi = 0$  this yields (2.12) and if  $\xi > 0$  and  $\gamma\psi > 0$  this relation cannot be solved analytically.

To derive the TECH-relation (2.8) for the neoclassical case, use (A.11), (A.2), and the appropriate parametrical restrictions. For the "Rebelo case," the TECH-relation (3.3) is derived by setting  $\beta = 1$  and substituting (A.9) and (A.1) (i.e.,  $Y/K = \alpha$ ) into (A.14). To derive the TECH-relation for the "Lucas case," differentiate (A.6) which yields  $\hat{\Theta}_1 + g - \hat{\Theta}_2 - \hat{h} = 0$  and substitute into this expression Eqs. (A.9), (A.12), (A.8), (A.6), (A.2), and (A.10) which yields  $(\vartheta - r) + g - [\vartheta - \hat{h} - (\epsilon - \hat{h} - \xi P)] - (g - \hat{L}) = 0$ . This implies

$$r = \epsilon + \hat{L} - \xi \left( \frac{A}{K} \right)^{-\gamma} . \quad (\text{A.16})$$

With  $\xi = 0$ , (A.16) yields relation (3.9). With  $\xi > 0$ , (A.16) can be used to eliminate  $A/K$  and (A.9) to eliminate  $Y/K$  which yields after substitution in (A.14)

$$g = \frac{r}{\beta} + \frac{1-\beta}{\beta} a - \frac{1}{\phi\gamma} \left\{ a^\mu - \xi(1-\beta) \frac{\gamma}{\vartheta} \left( \frac{r+a}{\beta} \right) a^{\psi\gamma} \right\} , \quad (\text{A.17})$$

$$\text{where } a \equiv \left( \frac{\xi}{\epsilon + \hat{L} - r} \right)^{1/\gamma} \left( = \frac{A}{K} \right) . \quad (\text{A.18})$$

<sup>8</sup> Formally the equality sign does not apply in the neoclassical case since  $u = 1$  will be binding. However, (A.13) is still valid because  $\xi = 0$  in the neoclassical case.



Equation (A.17) represents the TECH-relation between  $r$  and  $g$  as it is drawn in Fig. 4. Since only points at which  $C/K \geq 0$  and  $A/K \geq 0$  are meaningful,  $r$  has to be restricted to  $r^- \leq r \leq r^+$  where  $r^+ \equiv \epsilon + \hat{L}$  [from (A.18)] and  $r^-$  solves  $C/K = \{a^\mu - \xi(1 - \beta) \cdot (\gamma/\vartheta)[(r + a)/\beta]a^{\psi\gamma}\}/\phi\gamma = 0$  [from (A.13)]. From (A.17) it is clear that two TECH-lines drawn in a  $(r, g)$  plane for different values of  $\phi$  intersect at  $r^-$  and that the one with the higher value of  $\phi$  is above the one with the lower as long as the term between accolades in (A.17) is positive, i.e., as long as  $C/K \geq 0$ .

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